

Normal Distribution Questions

- 7 (a) The weight, X grams, of soup in a carton may be modelled by a normal random variable with mean 406 and standard deviation 4.2.

Find the probability that the weight of soup in a carton:

(i) is less than 400 grams; *(3 marks)*

(ii) is between 402.5 grams and 407.5 grams. *(4 marks)*

- (b) The weight, Y grams, of chopped tomatoes in a tin is a normal random variable with mean μ and standard deviation σ .

(i) Given that $P(Y < 310) = 0.975$, explain why:

$$310 - \mu = 1.96\sigma \quad \text{span style="float: right;">*(3 marks)*$$

(ii) Given that $P(Y < 307.5) = 0.86$, find, to two decimal places, values for μ and σ .

- 2 The heights of sunflowers may be assumed to be normally distributed with a mean of 185 cm and a standard deviation of 10 cm.

(a) Determine the probability that the height of a randomly selected sunflower:

(i) is less than 200 cm; *(3 marks)*

(ii) is more than 175 cm; *(3 marks)*

(iii) is between 175 cm and 200 cm. *(2 marks)*

- (b) Determine the probability that the mean height of a random sample of 4 sunflowers is more than 190 cm. *(4 marks)*
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6 When Monica walks to work from home, she uses either route A or route B.

- (a) Her journey time, X minutes, by route A may be assumed to be normally distributed with a mean of 37 and a standard deviation of 8.

Determine:

(i) $P(X < 45)$; *(3 marks)*

(ii) $P(30 < X < 45)$. *(3 marks)*

- (b) Her journey time, Y minutes, by route B may be assumed to be normally distributed with a mean of 40 and a standard deviation of σ .

Given that $P(Y > 45) = 0.12$, calculate the value of σ . *(4 marks)*

- (c) If Monica leaves home at 8.15 am to walk to work hoping to arrive by 9.00 am, state, with a reason, which route she should take. *(2 marks)*

- (d) When Monica travels to work from home by car, her journey time, W minutes, has a mean of 18 and a standard deviation of 12.

Estimate the probability that, for a random sample of 36 journeys to work from home by car, Monica's mean time is more than 20 minutes. *(4 marks)*

- (e) Indicate where, if anywhere, in this question you needed to make use of the Central Limit Theorem. *(1 mark)*
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- 7 (a) Electra is employed by E & G Ltd to install electricity meters in new houses on an estate. Her time, X minutes, to install a meter may be assumed to be normally distributed with a mean of 48 and a standard deviation of 20.

Determine:

(i) $P(X < 60)$; *(2 marks)*

(ii) $P(30 < X < 60)$; *(3 marks)*

(iii) the time, k minutes, such that $P(X < k) = 0.9$. *(4 marks)*

- (b) Gazali is employed by E & G Ltd to install gas meters in the same new houses. His time, Y minutes, to install a meter has a mean of 37 and a standard deviation of 25.

(i) Explain why Y is unlikely to be normally distributed. *(2 marks)*

(ii) State why \bar{Y} , the mean of a random sample of 35 gas meter installations, is likely to be approximately normally distributed. *(1 mark)*

(iii) Determine $P(\bar{Y} > 40)$. *(4 marks)*



Normal Distribution Answers

7 (a)	Weight, $X \sim N(406, 4.2^2)$				
(i)	$P(X < 400) = P\left(Z < \frac{400 - 406}{4.2}\right)$ $= P(Z < -1.428 \text{ to } -1.43)$ $= 1 - P(Z < 1.428 \text{ to } 1.43)$ $= 0.076 \text{ to } 0.077$	M1 m1 A1	3	Standardising (399.5, 400 or 400.5) with 406 and ($\sqrt{4.2}$, 4.2 or 4.2^2) and/or (406 - x) $\Phi(-z) = 1 - \Phi(z)$ AWRT 0.07636	
(ii)	$P(402.5 < X < 407.5) =$ $P(X < 407.5) - P(X < 402.5) =$ $P(Z < 0.36) - P(Z < -0.83)$ $= 0.64058 - (1 - 0.79673) = 0.433 \text{ to } 0.44$	M1 B2,1 A1	4	Difference OE AWRT; ignoring signs AWFW 0.43731	
(b)(i)	$0.975 \Rightarrow z = 1.96$ $P(Y < 310) = P\left(Z < \frac{310 - \mu}{\sigma}\right)$ <p>or</p> $x = \mu + / \pm z\sigma$	M1 M1		Accept explanation in words Standardising 310 using μ and σ Accept in words	
(ii)	Thus $\frac{310 - \mu}{\sigma} = 1.96 \Rightarrow$ result or $310 = \mu + 1.96\sigma \Rightarrow$ result NB: Working backwards from given equation \Rightarrow at most M1 M0 mo	m1	3	Equating AG Substitution	
(ii)	$0.86 \Rightarrow z = 1.08$ $310 - \mu = 1.96\sigma$ $307.5 - \mu = 1.08\sigma$ $2.5 = 0.88\sigma$ $\sigma = 2.84 \text{ to } 2.842$ $\mu = 304.4 \text{ to } 304.5$	B1 M1 A1 A1	4	AWRT 1.0803 Attempt at solving 2 equations each of form $x - \mu = z\sigma$ AWFW 2.841 AWFW 304.43	
Total			14		

2(a)	Height, $X \sim N(185, 10^2)$			
(i)	$P(X < 200) = P\left(Z < \frac{200-185}{10}\right)$ $= P(Z < 1.5)$ $= \Phi(1.5) = 0.933$	M1 A1 A1	3	standardising (199.5, 200 or 200.5) with 185 and $(\sqrt{10}, 10 \text{ or } 10^2)$ and/or $(185 - x)$ CAO; ignore sign AWRT (0.93319)
(ii)	$P(X > 175) = P\left(Z > \frac{175-185}{10}\right)$ $= P(Z > -1) = P(Z < 1)$ $= 0.841$	M1 m1 A1	3	standardising (174.5, 175 or 175.5) with 185 and $(\sqrt{10}, 10 \text{ or } 10^2)$ and/or $(185 - x)$ area change AWRT (0.84134)
(iii)	$P(175 < X < 200) = (i) - [1 - (ii)]$ $= 0.93319 - [1 - 0.84134]$ $= 0.774 \text{ to } 0.775$	M1 A1 \checkmark	2	or equivalent AWFW (0.77453) \checkmark on (i) and (ii) providing > 0
(b)	Mean of $\bar{X} = 185$	B1		CAO; may be implied by use in standardising
	Variance of $\bar{X} = \frac{10^2}{4} = 25$	B1		CAO; or equivalent
	$P(\bar{X} > 190) = P\left(Z > \frac{190-185}{5}\right)$ $= P(Z > 1) = 1 - \Phi(1)$ $= 0.159$	M1 A1 \checkmark	4	standardising 190 with 185 and 5 and/or $(185 - 190)$ AWRT (0.15866) \checkmark on (a)(ii) if used
		Total	12	

6(a)(i)	$P(X < 45) = P\left(Z < \frac{45-37}{8}\right)$ $= P(Z < 1)$ $= 0.841$	M1 A1 A1	3	Standardising (44.5, 45 or 45.5) with 37 and $(\sqrt{8}, 8 \text{ or } 8^2)$ and/or $(37 - x)$ CAO; ignore sign AWRT (0.84134)
(ii)	$P(30 < X < 45) = (i) - P(X < 30)$ $= (i) - P(Z < -0.875)$ $= (i) - [1 - (0.80785 \text{ to } 0.81057)]$ $= 0.648 \text{ to } 0.652$	M1 m1 A1	3	Used; OE Area change AWFW (0.65056)
(b)	$0.12 \Rightarrow z = 1.17 \text{ to } 1.18$ $z = \frac{45-40}{\sigma}$ $= 1.175$ $\sigma = 4.23 \text{ to } 4.28$	B1 M1 m1 A1	4	AWFW; ignore sign (1.1750) Standardising 45 with 40 and σ Equating z-term to z-value but not using 0.12, 0.88 or $ 1 - z $ AWFW

(c)	Route A: $P(X > 45) = 1 - (a)(i)$ Route B: $P(Y > 45) = 0.12$ so Monica should use Route B (smaller prob)	B1 ↑Dep↑ B1✓	2	OE; must use 45 ✓ on (a)(i); allow Route Y
(d)	Mean of $\bar{W} = 18$ Variance of $\bar{W} = \frac{12^2}{36} = 4$ $P(\bar{W} > 20) = P\left(Z > \frac{20-18}{2}\right)$ $= P(Z > 1) = 0.159$	B1 B1 M1 A1✓	4	CAO; can be implied by use in standardising CAO; OE Standardising 20 with 18 and 2 and/or (18 - 20) AWRT (0.15866); ✓ on (a)(i) if used
(e)	In part (d)	B1	1	CAO; OE
Total			17	

7(a)	Time, $X \sim N(48, 20^2)$			
(i)	$P(X < 60) = P\left(Z < \frac{60-48}{20}\right) =$ $P(Z < 0.6) = 0.725 \text{ to } 0.73$	M1 A1	2	Standardising (59.5, 60 or 60.5) with 48 and $(\sqrt{20}, 20 \text{ or } 20^2)$ and/or $(48 - x)$ AWFW (0.72575)
(ii)	$P(30 < X < 60) =$ $P(X < 60) - P(X < 30) =$ (i) - $P(X < 30) =$ (i) - $P(Z < -0.9) =$ (i) - $\{1 - P(Z < +0.9)\} =$ $0.72575 - \{1 - 0.81594\} =$ 0.54 to 0.542	M1 m1 A1	3	Difference or equivalent Standardising other than 60 and 30 \Rightarrow max of M1 m1 A0 Area change AWFW (0.54169)
(iii)	$0.9 \Rightarrow z = 1.28 \text{ to } 1.282$ $z = \frac{k-48}{20}$ $= 1.2816$ $k = 73.6 \text{ to } 74$	B1 M1 m1 A1	4	AWFW (1.2816) Standardising k with 48 and 20 Equating z -term to z -value; not using 0.9, 0.1, $ 1 - z $ or $\Phi(0.9) = 0.81594$ AWFW

(b)	Time, $Y \sim N(37, 25^2)$			
(i)	Use of $\mu - (2 \text{ or } 3) \times \sigma =$ $37 - (50 \text{ or } 75)$	M1		Or equivalent justification
	$< 0 \Rightarrow$ likely negative times	B1	2	for (likely) negative times
(ii)	Central Limit Theorem or n large / > 30	B1	1	
(iii)	Variance of $\bar{Y} = \frac{25^2}{35}$	B1		OE; stated or used
	$P(\bar{Y} > 40) = P\left(Z > \frac{40-37}{25/\sqrt{35}}\right) =$	M1		Standardising 40 with 37 and $25/\sqrt{35}$ and/or $(37 - 40)$
	$P(Z > 0.71) = 1 - P(Z < 0.71) =$	m1		Area change
	0.238 to 0.24	A1	4	AWFW (1 - 0.76115)
	Total		16	